High-Energy Quasielastic Proton-Proton Scattering and Final-State Interaction*

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The explanation by Drell and Hiida of the inelastic bump, observed by Cocconi *et al.*, in high-energy proton-proton scattering is shown to be inadequate for two reasons: (i) The $g(t)\equiv[d\sigma(s_1,t)/d\Omega]/[d\sigma(s_1,0)/d\Omega]$ chosen by Drell and Hiida gives too large wide-angle πN scattering, while a calculation consistent with wide-angle πN scattering gives a differential cross section smaller by a factor of five than the experimental cross section; (ii) the primary process in the Drell-Hiida mechanism gives much too small D- and F-wave amplitudes, so that even strong resonant final-state interactions in these states do not give any appreciable structure to the differential cross section. A formulation for the final-state interaction between the pion and the recoil nucleon is given, and it is shown that the nonresonant final-state scattering in S and P waves gives large enhancement.

I. INTRODUCTION

EXPERIMENTS done in CERN on p-p scattering above 10 GeV/c by Cocconi *et al.*[1,2] indicated the existence of an inelastic bump with two peaks. The rest mass of the recoiling system corresponding to these peaks coincided exactly with the second and third πN resonances which occur in $T=\frac{1}{2}$ states at energies 1.51 and 1.69 GeV, respectively. Feld and Iso[3] attempted to give an explanation of the bump in terms of a one-meson exchange diagram in which the target proton and the pion are left in an isobaric state. The objections against this explanation are: (i) The momentum transfers in these experiments are $\sim(1\text{ GeV}/c)^2$ so that this particular diagram is not expected to be the dominant process; (ii) The model predicts a peak corresponding to the (3,3) πN resonance which is not observed. Drell and Hiida[4] pointed out that these difficulties may be overcome by considering another one-meson exchange diagram in which the incident high-energy proton undergoes diffraction scattering on a virtual pion of the target proton. They succeeded in obtaining an inelastic bump and suggested that a final-state interaction between the pion and the recoil nucleon may explain the two peaks in the inelastic bump. The (3,3) resonance peak should not appear in this case, since the recoil nucleon and the pion formed from the target proton by diffraction scattering are expected to be in a $T=\frac{1}{2}$ state.

In the present work, we point out that two objections can be raised against the explanation of Drell and Hiida. The first objection is discussed in Sec. II. In Sec. III we discuss how the relative magnitudes of the partial wave amplitudes, due to the primary process, can be approximately evaluated. In Sec. IV we give a final-state interaction formulation for our present problem. In Sec. V we discuss our second objection using the results of Secs. III and IV. In Sec. VI, the main points are summarized, and some remarks about the final-state interaction formulation are made.

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[1] G. Cocconi, A. N. Diddens, E. Lillethum, and A. M. Wetherell, Phys. Rev. Letters **6**, 231 (1961).

[2] G. Cocconi, A. N. Diddens, E. Lillethum, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **7**, 450 (1961).

[3] B. T. Feld and C. Iso, Nuovo Cimento **21**, 59 (1961).

[4] S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961).

II. DIFFERENTIAL SCATTERING CROSS SECTION WITH NO FINAL-STATE INTERACTION

The diagram we are considering is shown in Fig. 1 (a). At the vertex A, we have πN diffraction scattering, as has been considered by Drell and Hiida. The box in the figure represents the final-state interaction, which we shall forget for the moment. The differential cross section in the lab system ($\mathbf{q}_i=0$) of the "primary" process[5] is ($\sum'$ means average over spins)

$$\begin{aligned}d\sigma_0=&\frac{p_{i0}}{p_i}(2\pi)^4\delta(p_i+q_i-p_f-k-q)\frac{m^2}{p_{i0}q_{i0}}\\&\times\sum_i{}'\sum_f|\langle k,p_f|\Gamma|p_i,\Delta\rangle|^2\\&\times\frac{d^3k}{(2\pi)^3 2\omega_k}\frac{m}{p_{f0}}\frac{d^3p_f}{(2\pi)^3}\frac{1}{(\Delta^2+\mu^2)^2}\\&\times\sum_i{}'\sum_f|\langle q|\Gamma_5|q_i,-\Delta\rangle|^2\frac{m}{q_0}\frac{d^3q}{(2\pi)^3}\\=&\frac{m}{p_i}\frac{1}{(2\pi)^5}|M(s_1,t)|^2\delta([p_i+q_i-p_f-q]^2+\mu^2)\\&\times\frac{m}{p_{f0}}d^3p_f\frac{1}{(\Delta^2+\mu^2)^2}\left(4\pi f^2\frac{\Delta^2}{\mu^2}\alpha\right)mqdq_0d(\cos\theta)d\phi\,,\quad(1)\end{aligned}$$

where

$$s_1=-(k+p_f)^2=-(p_i+\Delta)^2\,,\quad t=-(p_i-p_f)^2\,,$$

$$|M(s_1,t)|^2=\sum_i{}'\sum_f|\langle k,p_f|\Gamma|p_i,\Delta\rangle|^2\,,$$

[5] F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960); Phys. Rev. Letters **5**, 377 (1960); G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959). S. D. Drell, Rev. Mod. Phys. **33**, 458 (1961); E. Ferrari and F. Selleri, Suppl. Nuovo Cimento **24**, 453 (1962).

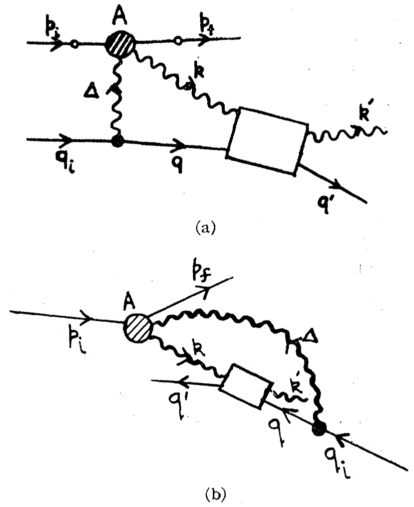

FIG. 1. (a) One-meson-exchange graph considered in the present paper, At the vertex A, diffraction scattering of the incident high-energy proton on a virtual pion of the target nucleon occurs. The box represents interaction in the final state between the pion and the recoil nucleon. (b) The same process in the center-of-mass system of the pion and the recoil nucleon.

and

$$\sum_i{}' \sum_f |\langle q|\Gamma_5|q_i, -\Delta\rangle|^2 = 4\pi f^2(\Delta^2/\mu^2)\alpha\,,$$

$$\alpha = 1 \quad \text{for} \quad \pi^0 \text{ emission}$$
$$= 2 \quad \text{for} \quad \pi^+ \text{ emission}.$$

We choose our z axis in the direction of the vector $(\mathbf{p}_i - \mathbf{p}_f)$ and take ϕ as the azimuthal angle of $\mathbf{q}$ relative to the plane containing $\mathbf{p}_i$ and $\mathbf{p}_f$. The integration over $d(\cos\theta)$ can now be carried out, and we arrive at the result

$$\frac{d^2\sigma_0}{dE_f d\Omega_f} = mp_f\left(4\pi\frac{f^2}{\mu^2}\alpha\right)\frac{m}{p_i}\frac{1}{(2\pi)^5}\frac{1}{4|\mathbf{p}_i-\mathbf{p}_f|} \times\int_{\Delta^2_{\min}}^{\Delta^2_{\max}}\frac{\Delta^2 d(\Delta^2)}{(\Delta^2+\mu^2)^2}\int_0^{2\pi}|M(s_1,t)|^2 d\phi\,, \quad (2)$$

with $\cos\theta$ determined by the δ-function;

$$\cos\theta = \{-t+\Delta^2+[(E_i-E_f)\Delta^2/m]+\mu^2\} \times(2q|\mathbf{p}_i-\mathbf{p}_f|)^{-1}. \quad (3)$$

Now, combining (3) with the restriction that $\cos^2\theta \leq 1$, we get the following inequality:

$$[-(t/m^2)-1-2(E_i-E_f)/m]\Delta^4 +\{4(E_i-E_f)^2-2t+2t(E_i-E_f)/m -2\mu^2-2\mu^2(E_i-E_f)/m\}\Delta^2-(-t+\mu^2)^2 \geq 0. \quad (4)$$

From inequality (4), $\Delta^2_{\max}$ and $\Delta^2_{\min}$ can be exactly worked out. To correct for the off-the-mass-shell effects at the vertices and in the pion propagator, we use the function $\phi(\Delta^2) = [1+(\Delta^2+\mu^2)/\alpha]^{-1}$ given by Ferrari and Selleri.[6] We also take into account the fact that the final pion can be π^+ as well as π^0. With these modifications, from Eq. (2), we get,

$$\frac{d^2\sigma_0}{dE_f d\Omega_f} = \frac{3}{2}\frac{1}{(2\pi)^4}\frac{f^2}{\mu^2}m^2\frac{p_f}{p_i}\frac{1}{|\mathbf{p}_i-\mathbf{p}_f|} \times\int_{\Delta^2_{\min}}^{\Delta^2_{\max}}\frac{\Delta^2 d(\Delta^2)}{(\Delta^2+\mu^2)^2}\phi^2(\Delta^2)\int_0^{2\pi}|M(s_1,t)|^2 d\phi. \quad (5)$$

The square of the matrix element $|M(s_1,t)|^2$ is related to the πN differential scattering cross section in the c.m. system by

$$|M(s_1,t)|^2 = \left(\frac{4\pi W}{m}\right)^2 \frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,t) \quad (s_1 = W^2). \quad (6)$$

If now $d\sigma/d\Omega_{\text{c.m.}}(s_1,t)$ is known as a function of s_1 and t, then inserting (6) in (5), we can evaluate $d^2\sigma_0/dE_f d\Omega_f$.

Drell and Hiida have made the following approximation here. They assume for high-energy diffraction scattering

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,t) = \frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,0)g(t)\,, \quad (7)$$

where the function $g(t)$ gives the dependence on the momentum transfer, t. Now if the amplitude $M(s_1,t)$ is completely imaginary for forward diffraction scattering, then

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,0) = \left(\frac{m}{4\pi W}\operatorname{Im}M(s_1,0)\right)^2. \quad (8)$$

From the optical theorem, $\operatorname{Im}M(s_1,0) = (kW/m)\sigma_T$, where k = c.m. momentum of πN system and σ_T = total cross section. From (7) they, therefore, arrive at the result

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,t) = \left(\frac{k}{4\pi}\right)^2 \sigma_T^2 g(t) = \frac{[s_1-(m+\mu)^2][s_1-(m-\mu)^2]}{4(4\pi)^2 s_1}\sigma_T^2 g(t). \quad (9)$$

Amati *et al.*[7] have derived the following t-dependence from the Mandelstam representation, making certain approximations:

$$g(t) = F^2|t|/4\mu^2;$$
$$F(x^2) = \ln[x+(1+x^2)^{1/2}]/x(1+x^2)^{1/2}. \quad (10)$$

Lovelace[8] recently has given the following formula for πN differential cross section at high energies as a

[6] E. Ferrari and F. Selleri, Phys. Rev. Letters **7**, 387 (1961).

[7] D. Amati, S. Fubini, A. Stanghellini, and M. Tonin, Nuovo Cimento **22**, 569 (1961).

[8] C. Lovelace, Nuovo Cimento **25**, 730 (1962).

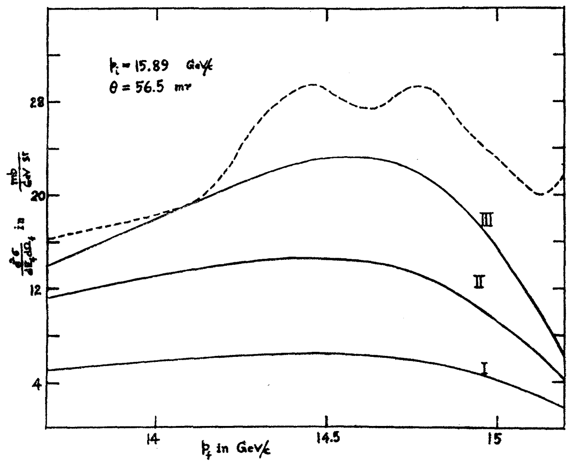

FIG. 2. Curve I—differential cross section using Lovelace dependence, but not final-state interaction. Curve II—differential cross section with Lovelace dependence and final-state interaction. Curve III—differential cross section using the dependence of Amati *et al.* The dashed curve represents the experimental results of Cocconi *et al.*

function of both of its variables:

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}}(s_1,t)=\alpha_0\left(\frac{k^2}{\mu^2}\right)\left(\frac{s_1}{\mu^2}\right)^{\gamma_0-\beta_0\eta^2}\ \text{mb/sr}\,, \tag{11}$$

where $\alpha_0=1.8138$, $\beta_0=2.70$, and $\gamma_0=-0.298$, and

$$\eta\approx -t/[2\mu+(-t+4\mu^2)^{1/2}]^2\,.$$

His formula agrees well with the wide-angle experiments, as well as with the forward peak and the Mandelstam representation.

We have calculated the differential cross section $d^2\sigma_0/dE_f d\Omega_f$ using the Amati *et al.* dependence [Eq. (10)], as well as the Lovelace dependence [Eq. (11)] for $p_i=15.89$ GeV/c and $\theta_{\text{lab}}=56.5$ mrad. Our results are shown in Fig. 2. The dashed curve represents the experimental result. Curves I and III give the differential cross section on the basis of the Drell-Hiida mechanism, i.e., the diffraction scattering of the incident proton by the pion cloud of the target nucleon. The distinction between I and III is that in I we have used the Lovelace dependence for πN diffraction scattering, while in III the dependence of Amati *et al.* has been used. The actual dependence assumed by Drell and Hiida in their work is $g(t)=(1-t/10\mu^2)^{-2}$ which is similar to that of Amati *et al.* and, in fact, 1.27 times larger in the relevant momentum transfer region ($-37.5\ \mu^2$ to $-40.6\ \mu^2$). This region lies considerably outside the πp main diffraction peak which extends up to $t=-27\mu^2$. The pion lab energy for the diffraction scattering at the vertex A varies between 2.5 and 9 GeV. The broad bump of curve III has been suggested by Drell and Hiida as the explanation of the experimental inelastic bump. However, for the large momentum transfers we are considering, the formula of Amati *et al.* gives a differential cross section 4 times larger than that given by the Lovelace formula. Since the Lovelace formula is in good agreement with the actual πN scattering results even for large t, so between curves I and III, only I can be considered as consistent with πN wide-angle scattering. As this curve is smaller by a factor of 5 times the experimental cross section, it means that the Drell-Hiida mechanism is inadequate to explain the bump.

However, we would like to mention that in the calculation of D-H a cutoff appears. This cutoff was adjusted to give the right magnitude of the bump corresponding to the $g(t)$ assumed. If now the integral over Δ^2 is taken to the maximum limit and the cutoff function $F(\Delta^2)$ is taken ≈ 1, then the height of the bump can be increased by a factor of 5.[4]

III. RELATIVE MAGNITUDES OF THE PARTIAL WAVES

In this section, we determine the relative magnitudes of the different partial waves due to the primary process in the system $\mathbf{k}+\mathbf{q}=0$. In this system, i.e., the c.m. system of the recoil nucleon and the pion produced, the process shown in Fig. 1(a) will look like that in Fig. 1(b). We denote all the quantities in the system $\mathbf{k}+\mathbf{q}=0$ by the suffix c. We take the direction of $\mathbf{q}_{ic}$ as the z axis, the plane containing $\mathbf{p}_{ic}$, $\mathbf{p}_{fc}$, and $\mathbf{q}_{ic}$ as the xz plane, and denote the polar angles of $\mathbf{q}_c$ by θ_c and ϕ_c. The invariant transition matrix element of the primary process can be written as

$$(p_f,k|\Gamma|p_i,\Delta)\frac{1}{\Delta^2+\mu^2}(q,\Delta|\Gamma_5|q_i)\phi(\Delta^2)\,. \tag{12}$$

$$\left(\phi(\Delta^2)=\frac{\alpha}{\Delta^2+\alpha+\mu^2};\quad \alpha\approx 60\mu^2\right).$$

In the $\mathbf{k}+\mathbf{q}=0$ system, we get

$$\Delta^2+\mu^2=2k_c q_{ic}(a_0-x)\,, \tag{13}$$

where

$$x=\cos\theta_c=\frac{\mathbf{q}_c\cdot\mathbf{q}_{ic}}{|\mathbf{q}_c|\,|\mathbf{q}_{ic}|}$$

and

$$a_0\equiv[2\omega_c(t+q_{ic}^2)^{1/2}-t]/(2k_c q_{ic})\,.$$

Similarly, we have

$$\Delta^2+\mu^2+\alpha=2k_c q_{ic}(a_3-x)\,, \tag{14}$$

where

$$a_3\equiv[2\omega_c(t+q_{ic}^2)^{1/2}-t+\alpha]/(2k_c q_{ic})\,;\quad \omega_c^2=(\mu^2+k_c^2)\,.$$

Both (13) and (14) are simple functions of $\cos\theta_c$.

The elements $(q,\Delta|\Gamma_5|q_i)$ and $(p_f,k|\Gamma|p_i,\Delta)$ depend not only on $\cos\theta_c$ but also on $\sin\theta_c$ and ϕ_c:

$$(q,\Delta|\Gamma_5|q_i)=-\frac{(E_{q_c}-m)^{1/2}(E_{q_{ic}}+m)^{1/2}}{2m}\chi^\dagger \times\left[(a_2-x)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} -(1-x^2)^{1/2}\begin{pmatrix}0 & e^{-i\phi_c}\\ e^{i\phi_c} & 0\end{pmatrix}\right]\chi_i, \quad (15)$$

$$a_2\equiv\frac{q_{ic}(E_{q_c}+m)}{q_c(E_{q_{ic}}+m)},\quad E_{q_c}=(q_c^2+m^2)^{1/2}.$$

$$\begin{aligned}(p_f,k|\Gamma|p_i,\Delta)&\approx \mathrm{Im}M(s_1,t)\,,\\ &\approx \mathrm{Im}M(s_1,0)[g(t)]^{1/2}\,,\\ &\approx (s_1-m^2)\frac{\sigma_T}{2m}[g(t)]^{1/2}\,,\\ &\approx [a_1-x-\tan\theta_{fc}\cos\phi_c(1-x^2)^{1/2}]\\ &\quad\times(-2p_{fc}k_c\cos\theta_{fc})\frac{\sigma_T}{2m}[g(t)]^{1/2}.\quad (16)\end{aligned}$$

In the derivation of (16), we have used

$$a_1\equiv-\frac{\mu^2+2(p_{fc})_0(k_c)_0}{2p_{fc}k_c\cos\theta_{fc}},$$

$$\mathrm{Im}M(s_1,0)=\frac{kW}{m}\sigma_T\approx(s_1-m^2)\frac{\sigma_T}{2m},$$

and

$$s_1=m^2+\mu^2+2p_{fc}k_c(\cos\theta_{fc}\cos\theta_c+\sin\theta_{fc}\sin\theta_c\cos\phi_c)+2(p_{fc})_0(k_c)_0$$

(θ_{fc} is the polar angle of $\mathbf{p}_{fc}$). From (12) and from (13)–(16), we get

$$(p_f,k|\Gamma|p_i,\Delta)\frac{1}{\Delta^2+\mu^2}(q,\Delta|\Gamma_5|q_i)\phi(\Delta^2) =\text{constant}\left[(T^{\mathrm{I}}+T^{\mathrm{II}}\cos\phi_c)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} +(T^{\mathrm{III}}+T^{\mathrm{IV}}\cos\phi_c)\begin{pmatrix}0 & e^{-i\phi_c}\\ e^{i\phi_c} & 0\end{pmatrix}\right],\quad (17)$$

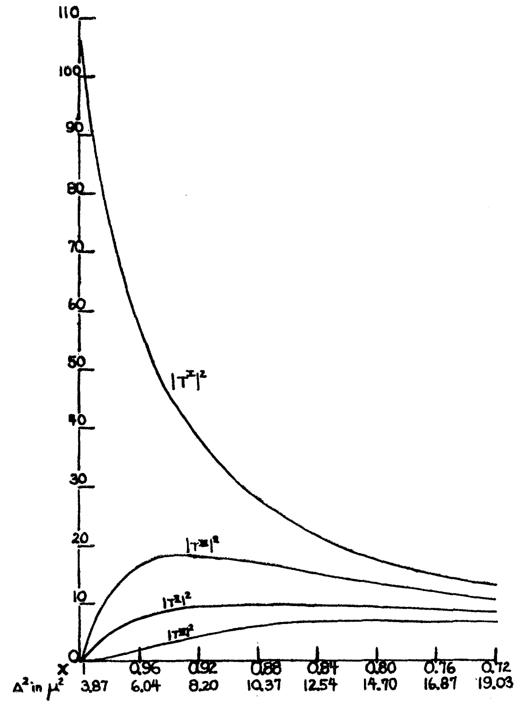

FIG. 3. Squares of the amplitudes T^{I}, T^{II}, T^{III}, and T^{IV} plotted against x and Δ^2. The values of a_0, a_1, a_2, and a_3 used are those given at the end of Sec. III.

where

$$T^{\mathrm{I}}\equiv\frac{(a_1-x)(a_2-x)}{(a_0-x)(a_3-x)};$$

$$T^{\mathrm{II}}\equiv-\frac{(a_2-x)\tan\theta_{fc}(1-x^2)^{1/2}}{(a_0-x)(a_3-x)};$$

$$T^{\mathrm{III}}\equiv-\frac{(a_1-x)(1-x^2)^{1/2}}{(a_0-x)(a_3-x)};$$

$$T^{\mathrm{IV}}\equiv\frac{\tan\theta_{fc}(1-x^2)}{(a_0-x)(a_3-x)}.$$

The "constant" in (17) is determined by the initial and the final lab momentum of the high-energy proton and the lab scattering angle. The parameters a_0, a_1, a_2, a_3 are also determined by these quantities.

In order to see the relative importance of the four amplitudes in (17), we have plotted their squares against x (equivalently, Δ^2) in Fig. 3. The contribution of these amplitudes to the single-pion-exchange process will be proportional to the area between the respective curves and the x axis, multiplied by a factor of 2π (for T^{I} and T^{III}) or π (for T^{II} and T^{IV}). The latter factor arises from integration over ϕ_c. There is no interference term between the direct and the spin-flip amplitudes when we take the sum and average of the final and initial spin states. Also, the interference between the first and second and between the third and fourth amplitudes vanishes when integrated over ϕ_c. Figure 3 shows that T^{I} is the dominating amplitude for low-momentum transfer. In our treatment of the final-state interaction, we shall approximate the primary amplitude by T^{I}. As we shall see later, this results in considerable simplification. On the basis of Fig. 3, we expect this to be a reasonable approximation.

We can now make the following Legendre polynomial expansion:

$$(a_1-x)\frac{1}{a_0-x}(a_2-x)\frac{1}{a_3-x}=\sum_l (2l+1)P_l(x)R_l \qquad (a_0 \text{ and } a_3>1). \tag{18}$$

From (17) and (18), we get

$$(p_f,k|\Gamma|p_i,\Delta)\frac{1}{\Delta^2+\mu^2}(q,\Delta|\Gamma_5|q_i)\phi(\Delta^2) \approx \text{constant} \sum_l (2l+1)P_l(x)R_l. \tag{19}$$

Equation (19) shows that the R_l's are the relative partial wave amplitudes of the primary process in the system $\mathbf{k}+\mathbf{q}=0$. From (18), we can work out the R_l's:

$$\begin{aligned} R_0&=[A_1-a_0A_2]+[1-a_0A_1+a_0^2A_2]Q_0(a_0),\\ R_1&=-\tfrac{1}{3}A_2+[1-a_0A_1+a_0^2A_2]Q_1(a_0),\\ R_2&=[1-a_0A_1+a_0^2A_2]Q_2(a_0),\\ R_3&=[1-a_0A_1+a_0^2A_2]Q_3(a_0); \end{aligned} \tag{20}$$

$$A_1\equiv\frac{a_1+a_2}{a_1a_2}-\frac{1}{a_3},\quad A_2\equiv\frac{1}{a_3^2}-\frac{a_1+a_2}{a_1a_2}\frac{1}{a_3}+\frac{1}{a_1a_2}.$$

The Q_l's in (20) are Legendre functions of the second kind. Knowing the parameters a_0, a_1, a_2, and a_3, we can find the numerical values of the partial wave amplitudes from (20). For $p_i=15.89$ GeV/c and $\theta_{\text{lab}}=56.5$ mrad, if we fix $p_{f,\text{lab}}=14.51$ GeV/c ($t=-39.02\mu^2$), we get $a_0=1.09$, $a_1=3.27$, $a_2=1.49$, $a_3=2.20$, and

$$\begin{aligned} R_0&=1.174, & R_0^2&=1.378,\\ R_1&=0.290, & 3R_1^2&=0.252,\\ R_2&=0.149, & 5R_2^2&=0.111,\\ R_3&=0.084, & 7R_3^2&=0.049. \end{aligned}$$

It will be noticed that the R_l's form a rapidly converging series, that R_2 and R_3 are small, and that the S-wave amplitude R_0 is by far the largest. We neglect partial wave amplitudes higher than R_3.

IV. FINAL-STATE INTERACTION FORMULATION

We shall now give a formulation for the final-state interaction between the pion produced and the recoil nucleon. Our notation will be similar to that of Watson.[9] We consider the total interaction potential as consisting of two parts: One is V, the primary production potential, and the other is v, the πN interaction potential. As in Watson, we denote the initial plane wave state by $|\chi_a)$ and the final plane wave state by $|\chi_B)$. A state $|\chi_B)$ will thus be, in our case, the product of a two-particle πN scattering state and a single-particle state of the diffraction scattered proton. We shall try to discuss our problem, as much as possible, in terms of scattering operators,[10] rather than in terms of the "unobservable" potentials.[11]

The total scattering operator is given by

$$\begin{aligned} t^+{}_{V+v}&=(V+v)\Omega^{(+)}\\ &=(V+v)\left[\Omega^{0(+)}+\frac{1}{E-H_0-V-v+i\epsilon}v\Omega^{0(+)}\right]\\ &\qquad\qquad [\text{Eq. (21) of Watson}]\\ &=t_V{}^++vGt_V{}^++t^-{}_{V+v}GvGt^+{}_V+v+t^-{}_{V+v}Gv, \end{aligned}\tag{21}$$

where

$$G\equiv\frac{1}{E-H_0+i\epsilon}\quad (E=E_B=E_a)$$

and

$$\begin{aligned} t^-{}_{V+v}&=\Omega^{(-)}(V+v)\\ &=\left[(V+v)\frac{1}{E-H_0-V-v+i\epsilon}+1\right](V+v). \end{aligned}$$

We are interested in the matrix element $(\chi_B|t_{V+v}{}^+|\chi_a)$. Since v is orthogonal to the initial state $|\chi_a)$ (v can be considered as P_BvP_B, where $P_B=\sum_B|\chi_B)(\chi_B|$ is the operator that projects out only states of the type B), we then get from (21)

$$\begin{aligned} (\chi_B|t_{V+v}{}^+|\chi_a)&=(\chi_B|[t_V{}^++(t_v{}^--t_v{}^-Gv)Gt_V{}^+\\ &\qquad+t_{V+v}{}^-GvGt_V{}^+]|\chi_a)\\ &\qquad\qquad \text{using}\quad t_v{}^-=v+t_v{}^-Gv\\ &=(\chi_B|\{[1+t_v{}^-G]t_V{}^+\\ &\qquad+(t_{V+v}{}^--t_v{}^-)GvGt_V{}^+\}|\chi_a). \end{aligned}\tag{22}$$

In terms of state functions, Eq. (22) can be written as

$$(\chi_B|t_{V+v}{}^+|\chi_a)=(\phi_B{}^{(-)}|V|\psi_a{}^{0(+)})+(\psi_B{}^{(-)}-\phi_B{}^{(-)}|v|\psi_a{}^{0(+)}) \tag{23}$$

where we have used

$$(\chi_B|(t_v{}^-G+1)=(\phi_B{}^{(-)}|,t_V{}^+|\chi_a)=V|\psi_a{}^{0(+)}).$$

Equation (23) above is Eq. (22) of Watson. The second term in (23) has been consistently neglected in the literature. Watson, however, has discussed under what conditions we can expect this term to be negligible:

[9] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
[10] G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).
[11] E. M. Ferreira, Ann. of Phys. (N. Y.) **16**, 235 (1961).

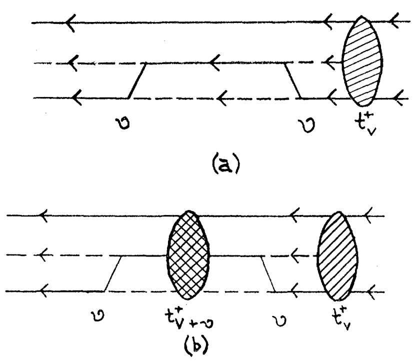

FIG. 4. Processes described by the amplitudes (a) $(\chi_B|vGvGt_V^+|\chi_a)$ and (b) $(\chi_B|vGt_{V+v}^+GvGt_V^+|\chi_a)$.

(i) if the primary potential V can be treated as a small perturbation, or (ii) if states of the type B do not give any important contribution. Neither of these conditions is satisfied in the peripheral process under our consideration. We, therefore, drop this assumption. Instead, we assume that the operator t_V^- of the production potential V does not give any scattering between two states of the type $|\chi_B)$ i.e., the matrix element of t_V^- between such states vanishes. This is obviously reasonable, because scattering between states of the type $|\chi_B)$ should be given by matrix elements of the operator t_v^-. (In these statements, we can equivalently write t_V^+ and t_v^+ instead of t_V^- and t_v^-).

Let us now examine the term $(\chi_B|t_{V+v}^-GvGt_V^+|\chi_a)$. We can write

$$t_{V+v}^- = t_V^- + t_V^-Gv(Gt_{V+v}^+ + 1) + v(Gt_{V+v}^+ + 1). \quad (24)$$

We notice that, from our above assumption, it follows a term of the form $(\chi_B|t_V^-Gv$ shall always vanish, since v can only produce states of the type $|\chi_B)$. Thus, we get

$$(\chi_B|t_{V+v}^-GvGt_V^+|\chi_a) = (\chi_B|v(Gt_{V+v}^+ + 1)GvGt_V^+|\chi_a). \quad (25)$$

The two amplitudes in (5) can be represented by the Figs. 4(a) and 4(b). We neglect the contribution due to these two amplitudes. The possibility of such complicated processes occurring through single potential terms is expected to be small. From (22) we, therefore, get

$$(\chi_B|t_{V+v}^+|\chi_a) = (\chi_B|[t_V^+ + t_v^-Gt_V^+ - t_v^-GvGt_V^+]|\chi_a). \quad (26)$$

The transition probability is

$$\begin{aligned}|(\chi_B|t_{V+v}^+|\chi_a)|^2 &= |(\chi_B|t_V^+|\chi_a)|^2 + |(\chi_B|t_v^-Gt_V^+|\chi_a)|^2 \\ &\quad + |(\chi_B|t_v^-GvGt_V^+|\chi_a)|^2 \\ &\quad + 2\,\mathrm{Re}\{(\chi_B|t_V^+|\chi_a)(\chi_B|vGt_V^+|\chi_a)^*\} \\ &\quad - 2\,\mathrm{Re}\{(\chi_B|t_v^-Gt_V^+|\chi_a)(\chi_B|t_v^-GvGt_V^+|\chi_a)^*\}. \quad (27)\end{aligned}$$

The amplitudes occurring in (27) can be represented in our peripheral process by the Figs. 5 (a), (b), (c), and (d). The interference terms in (27) correspond to interference between 5(a) and 5(b), and between 5 (c) and 5 (d).

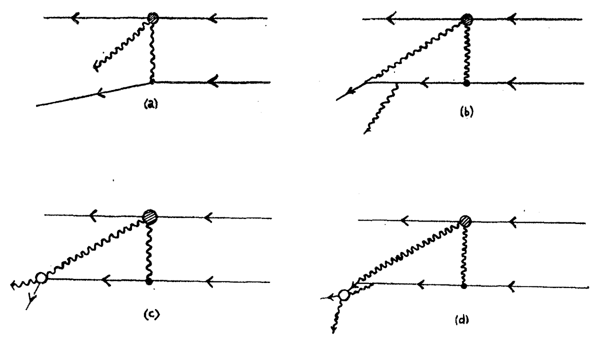

FIG. 5. Processes described by the amplitudes (a) $(\chi_B|t_V^+|\chi_a)$, (b) $(\chi_B|vGt_V^+|\chi_a)$, (c) $(\chi_B|t_v^-Gt_V^+|\chi_a)$ and (d) $(\chi_B|t_v^-GvGt_V^+|\chi_a)$.

The quantity we want to calculate is (see Fig. 1)

$$\int |S_{Ba}|^2 \frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3},$$

where the S-matrix element S_{Ba} is related to the transition amplitude $(\chi_B|t_{V+v}^+|\chi_a)$ by

$$\begin{aligned}S_{Ba} &= -i2\pi\delta(E_B-E_a)(\chi_B|t_{V+v}^+|\chi_a) \\ &= -i2\pi\delta(E_B-E_a)(\phi_B^{(-)}|V|\psi_a^{(+)}), \\ &\qquad \text{in the system} \quad \mathbf{p}_i+\mathbf{q}_i=0. \quad (28)\end{aligned}$$

Since the S-matrix element is Lorentz-invariant, we can define an invariant amplitude T_{Ba} in the following way:

$$S_{Ba} = -i(2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')T_{Ba}. \quad (29)$$

Writing

$$(\phi_B^{(-)}|V|\psi_a^{(+)}) = (2\pi)^3\delta^3(\mathbf{p}_f+\mathbf{k}'+\mathbf{q}')(\phi_B^{(-)}|V|\psi_a^{(+)})_r,$$

we get from (28) and (29)

$$(\phi_B^{(-)}|V|\psi_a^{(+)})_r = T_{Ba};$$

i.e., the reduced matrix element $(\phi_B^{(-)}|V|\psi_a^{(+)})_r$ is invariant. Thus, we obtain a Lorentz-invariant amplitude if we take out explicitly a momentum-conserving δ^3 function from the usual transition amplitude of the scattering theory. The new amplitude, so obtained, can be evaluated in any system we choose. One point to be remembered here is that in the formal scattering theory, the phase space integration is over the relative momentum vector only. However, if a momentum-conserving δ^3 function is taken out explicitly, then phase space integration over the total momentum vector has to be explicitly written. We shall find this discussion useful, because the amplitudes in (27) are all in the frame $\mathbf{p}_i+\mathbf{q}_i=0$, whereas we have to work in the system $\mathbf{p}_i+\mathbf{q}_i-\mathbf{p}_f=0$, i.e., the c.m. system of the pion and the recoil nucleon.

Now,

$$\int |S_{Ba}|^2 \frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3} = \int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q)|(\phi_B^{(-)}|V|\psi_a^{(+)})_r|^2 \frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}. \tag{30}$$

From (27), we find that the first term in (30) will be $[V|\psi_a^{0(+)})=t_V^+|\chi_a)]$

$$\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')|(\chi_B|V|\psi_a^{0(+)})_r|^2 \frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}$$
$$= \int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')|(\chi_B|V|\psi_a^{0(+)})_r|^2 \frac{d^3(k'+q')}{(2\pi)^3}\frac{d^3(\frac{1}{2}(k'-q'))}{(2\pi)^3}$$
$$= \frac{1}{(2\pi)^2}\int |(\chi_B|V|\psi_a^{0(+)})_r|^2 q'^2\left(\frac{dq'}{dE_{q'}}\right)d\Omega_{q'} \tag{31}$$

(evaluating in the system $\mathbf{p}_i+\mathbf{q}_i-\mathbf{p}_f=0$).

Now, $(\chi_B|V|\psi_a^{0(+)})_r=\sum_l (2l+1)P_l(\hat{q}_i\cdot\hat{q}')R_l$ in the system $\mathbf{p}_i+\mathbf{q}_i-\mathbf{p}_f=0$ [see Eq. (19)]. Using this expansion in (31), we get

$$\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')|(\chi_B|V|\psi_a^{0(+)})_r|^2 \frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3} = \sum_l (2l+1)|R_l|^2\frac{1}{\pi}q^2\left(\frac{dq}{dE_q}\right). \tag{32}$$

(In this case, the intermediate momenta k, q are exactly the same as the final momenta k', q'.)

From (27), again, we find that the second term in (30) will involve the amplitude

$$(\chi_B|t_v^-Gt_V^+|\chi_a)=(\phi|V|\psi_a^{0(+)}),$$

where

$$(\phi| \equiv (\phi_B^{(-)}|-(\chi_B|$$
$$\equiv (\chi_B|(t_v^-G+1)-(\chi_B|.$$

We first derive an expression for $(\phi|V|\psi_a^{0(+)})$, proceeding in the following way [using $U(0,t)_{t\to+\infty}|\chi_B)=|\phi_B^{(-)})$ and a complete set of states]:

$$(\phi_B^{(-)}|V|\psi_a^{0(+)}) = \sum_{\substack{n\\ t\to\infty}} (\chi_B|U(t,0)|\chi_n)(\chi_n|V|\psi_a^{0(+)}),$$
$$= \sum_{\substack{n\\ t\to\infty}} (\chi_B|e^{iH_0t}e^{-iHt}|\chi_n)(\chi_n|V|\psi_a^{0(+)}),$$
$$= \sum_{\substack{n\\ T\to\infty}} e^{i(E_B-E_n)T}(\chi_B|e^{iH_0T}e^{-iH2T}e^{-iH_0(-T)}|\chi_n)(\chi_n|V|\psi_a^{0(+)}), \quad (\text{here } T=\tfrac{1}{2}t)$$
$$= \sum_{\substack{n\\ T\to\infty}} e^{i(E_B-E_n)T}(\chi_B|U(T,-T)|\chi_n)(\chi_n|V|\psi_a^{0(+)}), \quad (\text{See Ref. 12})$$
$$= \sum_n [\delta(B-n)-2\pi i\delta(E_B-E_n)(\chi_B|v|\phi_n^{(+)})](\chi_n|V|\psi_a^{0(+)}),$$
$$= (\chi_B|V|\psi_a^{0(+)})-\sum_n [2\pi i\delta(E_B-E_n)(\chi_B|v|\phi_n^{(+)})(\chi_n|V|\psi_a^{0(+)})]. \tag{33}$$

From (33) we, therefore, get (changing from summation to integration; p_n denotes the relative momentum)

$$(\phi|V|\psi_a^{0(+)}) = \int -2\pi i\delta(E_B-E_n)(\phi_B^{(-)}|v|\chi_n)(\chi_n|V|\psi_a^{0(+)})d^3p_n/(2\pi)^3. \tag{34}$$

Equation (34) can be expressed in the following invariant way:

$$(\phi|V|\psi_a^{0(+)})_r = \int -i(2\pi)^4\delta^4(k'+q'-k-q)(\phi_B^{(-)}|v|\chi_n)_r(\chi_n|V|\psi_a^{0(+)})_r \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}. \tag{35}$$

In (35), k and q represent the momenta of the pion and the recoil nucleon in the intermediate state.

[12] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row, Peterson and Company, Evanston, Illinois, 1961).

The second term in (30) will now be

$$\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')|(\phi|V|\psi_a^{0(+)})_r|^2\frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}$$

$$=\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')\left|\int -i(2\pi)^4\delta^4(k'+q'-k-q)(\phi_B^{(-)}|v|\chi_n)_r(\chi_n|V|\psi_a^{0(+)})_r \times\frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\right|^2\frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}$$

$$=\int\left|\int -(\phi_B^{(-)}|v|\chi_n)_r(\chi_n|V|\psi_a^{0(+)})_r q^2\left(\frac{dq}{dE}\right)d\Omega_q\frac{1}{(2\pi)^2}\right|^2 q'^2\left(\frac{dq'}{dE'}\right)d\Omega_{q'}\frac{1}{(2\pi)^2}$$

$$=\int\left|\int f^{\alpha'\alpha}(\mathbf{q}',\mathbf{q})[\sum_l (2l+1)P_l(\hat{q}\cdot\hat{q}_i)R_l]\left(q\frac{dq}{dE}\right)^{1/2}\left(q'\frac{dq'}{dE'}\right)^{-1/2}q\frac{1}{2\pi}d\Omega_q\right|^2 q'^2\left(\frac{dq'}{dE'}\right)d\Omega_{q'}\frac{1}{(2\pi)^2}$$

$$\text{in the system}\quad \mathbf{p}_i+\mathbf{q}_i-\mathbf{p}_f=0. \qquad (36)$$

In (36), we have used the angular momentum expansion of the primary scattering amplitude and have introduced the πN scattering amplitude in the c.m. system given by

$$f^{\alpha'\alpha}(\mathbf{q}',\mathbf{q})=-\frac{1}{2\pi}(\mathbf{q}',\alpha'|t_v^{\pm}|\mathbf{q},\alpha)_r\left(q'\frac{dq'}{dE'}\right)^{1/2}\left(q\frac{dq}{dE}\right)^{1/2}.$$

The superscripts α and α' represent the initial and the final two-particle channels in the final state interaction. While α is always a πN channel, α' can be different because of the possibility of reaction. $f^{\alpha'\alpha}$ is related to the c.m. differential cross section by

$$\frac{d\sigma_{\alpha'\alpha}}{d\Omega_{\text{c.m.}}}=\frac{q'}{q}|f^{\alpha'\alpha}|^2.$$

Considering only the coherent amplitude,

$$f^{\alpha'\alpha}(\mathbf{q}',\mathbf{q})=\sum_l (2l+1)P_l(\hat{q}'\cdot\hat{q})f_l^{\alpha'\alpha}\qquad\left(f_l=\frac{1}{2l+1}[(l+1)f_{l^+}+lf_{l^-}]\right)$$

$$=4\pi\sum_{l,m} Y_{l,m}(\theta_{q'},\phi_{q'})Y_{l,m}^*(\theta_q,\phi_q)f_l^{\alpha'\alpha}. \qquad (37)$$

$$(\cos\theta_{q'}=\hat{q}'\cdot\hat{q}_i;\ \cos\theta_q=\hat{q}\cdot\hat{q}_i.)$$

Inserting this in (36), we have

$$\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')|(\phi|V|\psi_a^{0(+)})_r|^2\frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}$$

$$=\int\left|\int 4\pi\sum_{l,m} Y_{l,m}(\theta_{q'},\phi_{q'})Y_{l,m}^*(\theta_q,\phi_q)f_l^{\alpha'\alpha}\sum_{l'}(2l'+1)^{1/2}(4\pi)^{1/2}Y_{l',0}(\theta_q,\phi_q)R_{l'} \times\left(q\frac{dq}{dE}\right)^{1/2}\left(q'\frac{dq'}{dE'}\right)^{-1/2}q\frac{1}{2\pi}d\Omega_q\right|^2 q'^2\left(\frac{dq'}{dE'}\right)d\Omega_{q'}\frac{1}{(2\pi)^2}$$

$$=\int\left|4\pi\sum_l Y_{l,0}(\theta_{q'},\phi_{q'})(4\pi)^{1/2}(2l+1)^{1/2}f_l^{\alpha'\alpha}R_l\left(q\frac{dq}{dE}\right)^{1/2}\left(q'\frac{dq'}{dE'}\right)^{-1/2}q\frac{1}{2\pi}\right|^2 q'^2\left(\frac{dq'}{dE'}\right)d\Omega_{q'}\frac{1}{(2\pi)^2}$$

$$=\int|4\pi\sum_l (2l+1)P_l(\cos\theta_{q'})R_lf_l^{\alpha'\alpha}|^2 d\Omega_{q'}q^3\left(\frac{dq}{dE}\right)q'\frac{1}{(2\pi)^4}$$

$$=(4\pi)^2\cdot 4\pi\sum_l (2l+1)|R_l|^2|f_l^{\alpha'\alpha}|^2 q^3\left(\frac{dq}{dE}\right)q'\frac{1}{(2\pi)^4}$$

$$=\sum_l (2l+1)4qq'|f_l^{\alpha'\alpha}|^2|R_l|^2\frac{q^2}{\pi}\left(\frac{dq}{dE}\right). \qquad (38)$$

The third term in (30) will involve the square of the amplitude shown in Fig. 5(d). As is obvious from the diagrams, this term should be negligible compared to the sum of the squares of the amplitudes shown in 5(a) and 5(c). In the Appendix, we shall present arguments showing that the interference terms in (27) should vanish when integrated over the direction of the relative momentum of the final outgoing interacting pair. The right-hand side of (30) will, therefore, be the sum of the two terms, viz. (32) and (38).

The differential cross section with final-state interaction is now given by

$$\frac{d^2\sigma}{dE_f d\Omega_f}=\frac{E_i}{p_i}\int (2\pi)^4\delta^4(p_i+q_i-p_f-k'-q')\,|(\phi_B{}^{(-)}|V|\psi_a{}^{(+)})_r|^2\frac{d^3k'}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}p_f{}^2\left(\frac{dp_f}{dE_f}\right)\frac{1}{(2\pi)^3}$$

$$=\frac{E_i}{p_i}\sum_l (2l+1)|R_l|^2[1+4\sum_{\alpha'} q'q|f_l{}^{\alpha'\alpha}|^2]\frac{q^2}{\pi}\left(\frac{dq}{dE}\right)p_f{}^2\left(\frac{dp_f}{dE_f}\right)\frac{1}{(2\pi)^3}. \quad (39)$$

From (32), we can also calculate the differential cross section with no final-state interaction, which is

$$\frac{d^2\sigma_0}{dE_f d\Omega_f}=\frac{E_i}{p_i}\sum_l (2l+1)|R_l|^2\frac{1}{\pi}q^2\left(\frac{dq}{dE}\right)p_f{}^2\left(\frac{dp_f}{dE_f}\right)\frac{1}{(2\pi)^3}. \quad (40)$$

In (39) and (40), p_i and p_f are the initial and the final lab momenta of the high-energy proton, and E_i and E_f are the corresponding lab energies.

From (39) and (40), we get

$$\frac{d^2\sigma}{dE_f d\Omega_f}=\frac{d^2\sigma_0}{dE_f d\Omega_f}\left[1+\frac{1}{Q}\sum_{l=0}^{\infty}(2l+1)|R_l|^2(\sum_{\alpha'} 4q'q|f_l{}^{\alpha'\alpha}|^2)\right], \quad (41)$$

where

$$Q\equiv\sum_{l=0}^{\infty}(2l+1)|R_l|^2$$

and

$$f_l{}^{\alpha'\alpha}\equiv\frac{1}{2l+1}[(l+1)f_{l^+}{}^{\alpha'\alpha}+lf_{l^-}{}^{\alpha'\alpha}].$$

The quantity inside the square brackets in (41) gives the final-state enhancement factor. However, in (41) we have considered only the coherent amplitude in the final πN interaction. If the spin-flip amplitude is also taken, and we sum over final spin states and average over initial spin states, then there is a further term inside the square brackets in (41), and we have,

$$\frac{d^2\sigma}{dE_f d\Omega_f}=\frac{d^2\sigma_0}{dE_f d\Omega_f}\left[1+\frac{1}{Q}\sum_{l=0}^{\infty}|R_l|^2\frac{1}{(2l+1)}\sum_{\alpha'} 4q'q|(l+1)f_{l^+}{}^{\alpha'\alpha}+lf_{l^-}{}^{\alpha'\alpha}|^2\right.$$
$$\left.+\frac{1}{Q}\sum_{l=1}^{\infty}|R_l|^2\frac{(l+1)!}{(2l+1)(l-1)!}\sum_{\alpha'} 4q'q|f_{l^+}{}^{\alpha'\alpha}-f_{l^-}{}^{\alpha'\alpha}|^2\right]. \quad (42)$$

V. DIFFERENTIAL SCATTERING CROSS SECTION WITH FINAL-STATE INTERACTION

Using the numerical values of the relative amplitudes R_l, given at the end of Sec. III, we shall now show that the contribution to the enhancement factor (E.F.) due to D and F waves is too small, so that even resonant final-state interactions in these states will not give any appreciable structure to the differential cross section. We shall also show that the S- and P-wave πN final state interaction will give a large enhancement to the differential cross section.

The contribution to the enhancement factor (E.F.) in (42) by a partial wave ($l>0$) is

$$\frac{4|R_l|^2}{Q}\sum_{\alpha'}\left\{\frac{1}{(2l+1)}q'q|(l+1)f_{l^+}{}^{\alpha'\alpha}+lf_{l^-}{}^{\alpha'\alpha}|^2\right.$$
$$\left.+\frac{(l+1)!}{(2l+1)(l-1)!}q'q|f_{l^+}{}^{\alpha'\alpha}-f_{l^-}{}^{\alpha'\alpha}|^2\right\}. \quad (43)$$

A resonant partial wave amplitude $f_{l,J}{}^{\alpha'\alpha}$ is given by

$$f_{l,J}{}^{\alpha'\alpha}=\frac{1}{(qq')^{1/2}}\frac{(\frac{1}{2}\Gamma_\alpha)^{1/2}(\frac{1}{2}\Gamma_{\alpha'})^{1/2}}{(E_r-E)-\frac{1}{2}i\Gamma_T}, \quad (44)$$

where $\frac{1}{2}\Gamma_\alpha$, $\frac{1}{2}\Gamma_{\alpha'}$ are the half-widths in the channels α, α'; $\frac{1}{2}\Gamma_T=\sum_{\alpha'}\frac{1}{2}\Gamma_{\alpha'}$ is the total half-widths; q, q' are the relative momenta in the channels α and α'; α, in our case, is the πN channel.

For $l=2$, if we disregard the nonresonant amplitude, then the contribution to the E.F. due to the second πN resonance $D_{3/2}$ ($T=\frac{1}{2}$) is

$$\frac{4|R_2|^2}{Q}2\frac{(\frac{1}{2}\Gamma_\alpha)(\frac{1}{2}\Gamma_T)}{(E_r-E)^2+(\frac{1}{2}\Gamma_T)^2}<\frac{4|R_2|^2}{Q}2. \tag{45}$$

Similarly, for $l=3$, disregarding the nonresonant amplitude, the contribution to the E.F. due to the third πN resonance $F_{5/2}$ ($T=\frac{1}{2}$) is

$$\frac{4|R_3|^2}{Q}3\frac{(\frac{1}{2}\Gamma_\alpha)(\frac{1}{2}\Gamma_T)}{(E_r-E)^2+(\frac{1}{2}\Gamma_T)^2}<\frac{4|R_3|^2}{Q}3. \tag{46}$$

To estimate the contribution to the E.F. due to nonresonant S- and P-wave πN interaction, we make the following simple optical-model approximation. We assume

$$f_{l,J}{}^{\alpha\alpha}=\frac{e^{2i\delta_{lJ}}-1}{2iq}=\frac{\rho-1}{2iq},$$

where ρ is a real parameter. Each of the S and P waves now contribute a term of the following form to the E.F.:

$$\begin{aligned}\frac{1}{Q}(2l+1)|R_l|^2 4q^2|f_l{}^{\alpha\alpha}|^2 &\\ +\frac{1}{Q}(2l+1)|R_l|^2\sum_{\alpha'(\neq\alpha)}4q'q|f_l{}^{\alpha'\alpha}|^2 &\\ =\frac{1}{Q}(2l+1)|R_l|^2\{(\rho-1)^2+(1-\rho^2)\}. &\end{aligned} \tag{47}$$

In deriving (47), we have used the relation

$$\begin{aligned}\sigma_{\text{in}}=\sum_{\alpha'(\neq\alpha)}\sigma_{\alpha'\alpha}&=\sum_l(\Pi/q^2)\,(2l+1)\sum_{\alpha'(\neq\alpha)}4qq'|f_l{}^{\alpha'\alpha}|^2\\ &=\sum_l(\Pi/q^2)\,(2l+1)(1-\rho_l{}^2).\end{aligned}$$

We have taken the value $\rho=0.3$ for the S- and P-wave πN scattering. This value gives a nonresonant total cross section of 33 mb for the second pion-nucleon resonance, and is consistent with experimental cross sections. The S- and P-wave contributions to the E.F. is now, from (47)

$$\frac{R_0{}^2+3R_1{}^2}{Q}(2-2\rho)=\frac{R_0{}^2+3R_1{}^2}{Q}1.4. \tag{48}$$

Using the numerical values given at the end of Sec. III for the R_l's, we get

$$Q=1.790,\quad (R_0{}^2+3R_1{}^2)1.4/Q=1.275$$
$$4R_2{}^2/Q=0.050,\qquad 4R_3{}^2/Q=0.016.$$

Thus, we see that the E.F., with only S- and P-wave πN interaction included, is equal to 2.275. From (45) and (46) we find that the contributions of $D_{3/2}$ and $F_{5/2}$ resonances to this factor are less than 0.1 and 0.048, respectively. Both these numbers are negligible compared with 2.275. This essentially means that though the D- and F-wave contributions have resonant forms, yet they will give little structure to the differential cross section, because their magnitudes are too small.

The differential cross section with final-state interaction in S and P waves is given by curve II in Fig. 2. Comparing it with curve I, which gives the differential cross section with no final-state interaction, we notice that the S- and P-wave nonresonant πN interaction gives a large enhancement (the main contribution comes, of course, from the S wave).

VI. DISCUSSION

We have shown that a Drell-Hiida calculation consistent with πN wide-angle scattering is smaller by a factor of 5 than the experimental differential cross section. We have considered the experiment done at $p_i=15.89$ GeV/c and $\theta_{\text{lab}}=56.5$ mrad by Cocconi *et al.* The bump obtained by Drell-Hiida, in apparent agreement with the experimental result, was pointed out to be due to the $g(t)$ chosen by them which gives too large πN wide-angle scattering. We also pointed out that the Drell-Hiida explanation is insufficient to explain the fine structure of the experimental bump, because the D- and F-wave amplitudes due to the primary process are too small; and therefore, even strong resonant πN final-state interactions give little structure. We, thus, conclude that a simple peripheral calculation, as done at present, is not able to explain the inelastic bump in proton-proton scattering above 10 GeV/c. In this context, it is worth mentioning that a different explanation, based on the idea of Regge pole exchanges, has been suggested by Frautschi, Gell-Mann, and Zachariasen.[13]

In Sec. IV we pointed out why we dropped the usual assumption in final-state interaction,[14] viz. $(\phi_B{}^{(-)}|V|\psi_a{}^{(+)})\approx(\phi_B{}^{(-)}|V|\psi_a{}^{0(+)})$. Instead, we as-

[13] S. C. Frautschi, M. Gell-Mann and F. Zachariasen, Phys. Rev. **126**, 2204 (1962). V. N. Gribov, B. L. Ioffe, I. Ya Pomeranchuk, and A. P. Rudik, Zh. Eksperim. i Teor. Fiz. **42**, 1260 (1962) [translation: Soviet Phys.—JETP **15**, 984 (1962)]; A. P. Contogouris, S. C. Frautschi, and How-sen Wong, Phys. Rev. **129**, 974 (1963). We shall like to point out that by using the Lovelace dependence for πN diffraction scattering, we have actually taken into account the exchange of a "Pomeranchuk" Regge pole in t variable.

[14] For dispersion theoretic formulation of final-state interaction and its equivalence with formal scattering theory treatment, see M. Jacob, G. Mahoux and R. Omnes, Nuovo Cimento **23**, 838 (1962); J. D. Jackson, Nuovo Cimento **25**, 1038 (1962).

sumed that the scattering operator t_V^+ of the primary production potential does not give any scattering between states of the type $|\chi_B)$. Our assumption was based on the argument that scattering between states of the type $|\chi_B)$ should only be given by the operator t_v^+ of the πN interaction potential v. We shall also like to note here some other points of difference. Watson, in his final-state interaction formulation, assumed two conditions: (i) The primary interaction should be of practically zero range, (ii) It should be attractive, so that the two interacting particles tend to stick together. Neither of these appear in our present case, because of the special way we consider the whole process occurs. Here, the incident high-energy proton is diffraction scattered by a virtual pion of the target nucleon cloud, leaving behind the real pion and the target nucleon, which will then interact strongly with each other. Thus, even though the range of the primary process is of the order $\mu^{-1}=1.4$ F, and the pion-nucleon phase shifts may not all be positive, it is sensible to think of the whole process as divided into primary production followed by a final-state interaction of the pion and the nucleon.

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APPENDIX

We shall now present arguments showing that the interference terms in Eq. (27) should vanish when integrated over the direction of the relative momentum of the final interacting pair.

Let us consider the amplitude $(\chi_B|vGt_V^+|\chi_a)$ occurring in the first interference term in (27). This amplitude describes the process shown in Fig. 5(b). We can write it in the following way:

$$(\chi_B|vGt_V^+|\chi_a)=\int(\chi_B|v|\chi_n)\left[P\frac{1}{E-E_n}-i\pi\delta(E-E_n)\right](\chi_n|t_V^+|\chi_a)\frac{d^3p_n}{(2\pi)^3},\quad (E=E_B=E_a) \tag{A1}$$

The principal value part corresponds to contribution from energy-nonconserving intermediate states. Now, the only intermediate states which should come into our consideration are the energy-conserving ones, because the pion in the intermediate states, produced by the primary process, is taken as areal pion. We, therefore, neglect the principal part in (A1). This leads to

$$(\chi_B|vGt_V^+|\chi_a)=-\frac{i\pi}{(2\pi)^3}\int(\chi_B|v|\chi_n)(\chi_n|t_V^+|\chi_a)p_n^2\left(\frac{dp_n}{dE_n}\right)d\Omega_n. \tag{A2}$$

We shall work in the system $\mathbf{p}_i+\mathbf{q}_i-\mathbf{p}_f=0$ and shall take the direction of $\mathbf{q}_i$ as the z axis, as in Sec. II. In this system, we have

$$(\chi_n|t_V^+|\chi_a)=\sum_l(2l+1)P_l(\cos\theta_n)R_l,\quad(\cos\theta_n=\hat{q}_i\cdot\hat{p}_n)\quad[\text{see Eq. (19)}]. \tag{A3}$$

We shall now define an amplitude

$$f^0(\mathbf{p}_B,\mathbf{p}_n)=-\frac{1}{2\pi}(\chi_B|v|\chi_n)p_n\left(\frac{dp_n}{dE_n}\right) \tag{A4}$$

similar to the πN elastic scattering amplitude

$$f(\mathbf{p}_B,\mathbf{p}_n)=-\frac{1}{2\pi}(\chi_B|t_v^+|\chi_n)p_n\left(\frac{dp_n}{dE_n}\right). \tag{A5}$$

The difference between (A4) and (A5) is that in the former, the matrix element of the potential v occurs, instead of that of the operator t_v^+. Now, $f(\mathbf{p}_B,\mathbf{p}_n)$ has the following expansion (considering only the coherent amplitude):

$$\begin{aligned}f(\mathbf{p}_B,\mathbf{p}_n)&=\sum_l(2l+1)P_l(\hat{p}_B\cdot\hat{p}_n)f_l\\&=4\pi\sum_{l,m}Y_{l,m}(\theta_B,\phi_B)Y_{l,m}^*(\theta_n,\phi_n)f_l\quad[\text{see Eq. (37)}].\end{aligned} \tag{A6}$$

We assume a similar expansion for $f^0(\mathbf{p}_B,\mathbf{p}_n)$, viz.

$$f^0(\mathbf{p}_B,\mathbf{p}_n)=4\pi\sum_{l,m} Y_{l,m}(\theta_B,\phi_B)Y_{l,m}^*(\theta_n,\phi_n)f_l^0. \qquad \text{(A7)}$$

From (A2), we now have

$$\chi_B|vGt_V^+|\chi_a)=\frac{i\pi}{(2\pi)^2}\int p_n f^0(\mathbf{p}_B,\mathbf{p}_n)(\chi_n|t_V^+|\chi_a)d\Omega_n \quad [\text{using (A4)}]$$

$$=\frac{i\pi}{(2\pi)^2}\int p_n[\sum_{l',m'} 4\pi Y_{l',m'}(\theta_B,\phi_B)Y_{l',m'}^*(\theta_n,\phi_n)f_{l'}^0][\sum_l (2l+1)^{1/2}(4\pi)^{1/2}Y_{l,0}(\theta_n,\phi_n)R_l]d\Omega_n$$

$$[\text{from (A3) and (A7)}]$$

$$=i\sum_l (2l+1)P_l(\cos\theta_B)(p_Bf_l^0)R_l. \qquad \text{(A8)}$$

Next we consider the amplitude $(\chi_B|t_v^-Gt_V^+|\chi_a)$ which describes the process in Fig. 5(c). From Eq. (34), we have

$$(\chi_B|t_v^-Gt_V^+|\chi_a)=(\phi|V|\psi_a^{0(+)})$$

$$=-\frac{i}{(2\pi)^2}\int(\phi_B^{(-)}|v|\chi_n)(\chi_n|V|\psi_a^{0(+)})p_n^2\left(\frac{dp_n}{dE_n}\right)d\Omega_n$$

$$=\frac{i}{2\pi}\int f(\mathbf{p}_B,\mathbf{p}_n)p_n(\chi_n|t_V^+|\chi_a)d\Omega_n \quad [\text{using (A5)}]$$

$$=2i\sum_l (2l+1)P_l(\cos\theta_B)(p_nf_l)R_l \quad [\text{using (A3) and (A6)}]. \qquad \text{(A9)}$$

Let us now consider the amplitude $(\chi_B|t_v^-GvGt_V^+|\chi_a)$ occurring in the second interference term in (27). This amplitude describes the process shown in Fig. 5(d). We write this amplitude in the following way:

$$(\chi_B|t_v^-GvGt_V^+|\chi_a)=\sum_n (\chi_B|t_v^-G|\chi_n)(\chi_n|vGt_V^+|\chi_a)$$

$$=\int(\chi_B|t_v^-G|\chi_n)[i\sum_l (2l+1)P_l(\cos\theta_n)p_nf_l^0R_l]\frac{d^3p_n}{(2\pi)^3} \quad [\text{using Eq. (A8)}]. \qquad \text{(A10}$$

Now,

$$(\chi_B|(t_v^-G+1)|\chi_n)=(\phi_B^{(-)}|\chi_n)$$

$$=(\chi_B|U(t,0)|\chi_n)$$

$$t\rightarrow+\infty$$

$$=\delta(B-n)-2\pi i\delta(E_B-E_n)(\chi_B|v|\phi_n^{(+)}) \quad [\text{see the derivation of Eq. (33)}]. \qquad \text{(A11)}$$

Therefore, from (A10)

$$(\chi_B|t_v^-GvGt_V^+|\chi_a)=\int -2\pi i\delta(E_B-E_n)(\chi_B|v|\phi_n^{(+)})[i\sum_l (2l+1)P_l(\cos\theta_n)p_nf_l^0R_l]\frac{d^3p_n}{(2\pi)^3}$$

$$=-\frac{i}{(2\pi)^2}\int(\chi_B|v|\phi_n^{(+)})[i\sum_l (2l+1)P_l(\cos\theta_n)p_nf_l^0R_l]p_n^2\left(\frac{dp_n}{dE_n}\right)d\Omega_n$$

$$=-2\sum_l (2l+1)P_l(\cos\theta_B)(p_nf_l)(p_nf_l^0)R_l \quad [\text{using (A5) and (A6)}]. \qquad \text{(A12)}$$

We should like to say a word here about the partial wave amplitudes f_l^0. As seen from Eqs. (A6) and (A7), they are determined by the potential v. In relativistic theories, we say that the potential is given by left-hand discontinuities. Since the partial waves in the physical region due to the left-hand discontinuities are always real, therefore, we shall argue that the f_l^0's are all real. Intuitively, one can think of the f_l^0's as some sort of Born amplitudes.

We consider the first interference term in (27) integrated over the direction $\mathbf{p}_B$ (the relative momentum of the outgoing pair):

$$2\int \mathrm{Re}\{(\chi_B|t_V{}^+|\chi_a)(\chi_B|vGt_V{}^+|\chi_a)^*\}d(\cos\theta_B)$$

$$=2\,\mathrm{Re}\int[\sum_l (2l+1)P_l(\cos\theta_B)R_l][i\sum_{l'} (2l'+1)P_{l'}(\cos\theta_B)p_B f_{l'}{}^0 R_{l'}]^* d(\cos\theta_B) \quad [\text{using (A3) and (A8)}],$$

$$=2\,\mathrm{Re}[-i2\sum_l (2l+1)|R_l|^2(p_B f_l{}^0)] \quad (f_l{}^0\text{'s are real}),$$

$$=0.$$

Next, we consider the other interference term,

$$2\int \mathrm{Re}\{(\chi_B|t_v{}^-Gt_V{}^+|\chi_a)(\chi_B|t_v{}^-GvGt_V{}^+|\chi_a)^*\}d(\cos\theta_B)$$

$$=2\,\mathrm{Re}\int[2i\sum_l (2l+1)P_l(\cos\theta_B)(p_n f_l)R_l][-2\sum_{l'} (2l'+1)P_{l'}(\cos\theta_B)(p_n f_{l'})(p_n f_{l'}{}^0)R_{l'}]^* d(\cos\theta_B)$$

$$=2\,\mathrm{Re}[-i8\sum_l (2l+1)|R_l|^2 p_n{}^2|f_l|^2(p_n f_l{}^0)]$$

$$=0.$$

Thus, the two interference terms in (27) vanish when integrated over the direction of $\mathbf{p}_B$.

For the processes shown in Figs. 5 (c) and 5 (d), we can also have inelastic scattering and spin-flip scattering in the final πN interaction. However, in these cases too, the interference terms between these two graphs can be shown to vanish when the integration over $\mathbf{p}_B$ is carried out.